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Risk aversion and risk seeking in multicriteria forest management: a Markov decision process approach

Joseph Buongiorno, Mo Zhou, and Craig Johnston

Abstract: Markov decision process models were extended to reflect some consequences of the risk attitude of forestry decision makers. One approach consisted of maximizing the expected value of a criterion subject to an upper bound on the variance or, symmetrically, minimizing the variance subject to a lower bound on the expected value. The other method used the certainty equivalent criterion, a weighted average of the expected value and variance. The two approaches were applied to data for mixed softwood–hardwood forests in the southern United States with multiple financial and ecological criteria. Compared with risk neutrality or risk seeking, financial risk aversion reduced expected annual financial returns and production and led to shorter cutting cycles that lowered the expected diversity of tree species and size, stand basal area, stored CO_2e, and old-growth area.

Key words: uneven-aged management, risk aversion, economics, multiple criteria, Markov decision process, quadratic programming.

Résumé : Des modèles de décision de Markov ont été développés pour présenter quelques conséquences de l'attitude des décideurs en foresterie face au risque. Une des approches consistait à maximiser la valeur prévue d'un critère assujetti à la valeur maximale de l'échelle de variance, ou de manière symétrique, de minimiser la variance assujettie à la limite inférieure de la valeur prévue. L'autre méthode utilisait comme critère l'équivalent certain, soit une moyenne pondérée de la valeur prévue et de sa variance. Les deux approches furent appliquées à des données issues de forêts mixtes de feuillus et de conifères du sud des États-Unis en prenant en compte plusieurs critères financiers et écologiques. Comparativement à la neutralité à l'égard du risque ou à la recherche du risque, l'aversion pour le risque financier réduit les rendements financiers annuels attendus ainsi que la production, ce qui mène à des cycles de récolte plus courts causant une diminution de la diversité et de la taille des arbres, de la surface terrière, de la séquestration d'éq. CO_2 et des secteurs de vieilles forêts attendus. [Traduit par la Rédaction]

Mots-clés : aménagement inéquienne, aversion pour le risque, économie, critères multiples, processus décisionnel de Markov, programmation quadratique.

Introduction

Uncertainty about the future affects forestry decisions due to potential losses or gains associated with economic fluctuations, climate change, and attendant disturbances. Accordingly, there is an expanding literature on the role of risk and uncertainty in forest management (Kant and Alavalapati 2014, pp. 307–369), including the risk preferences of private landowners (Andersson and Gong 2010; Lönnstedt and Svensson 2000) and of the public towards forest resources (Ananda and Herath 2005).

Most of this literature has dealt with the management of even-aged, single-species forests for which a common message is that optimal decisions assuming risk neutrality become suboptimal with differing risk attitudes. In particular, financial risk aversion leads to shorter rotations (Caulfield 1988; Couture and Reynaud 2008), a finding that appears to be robust across methodologies (Pukkala and Kangas 1996; Gong 1998; Alvarez and Koskela 2006). Ollikainen (1990) also shows that, facing uncertain interest rates, forest owners who are lenders tend to harvest less than borrowers, and according to Gong and Löfgren (2003), lenders' decisions depend on the variance of future wealth. Risk preferences affect other aspects of forestry, including land use (Parks 1995; Delacote 2007) and the diversification of age classes and stand structures (Tahvonen and Kallio 2006; Roessiger et al. 2011).

This paper deals with the management of complex, uneven-aged, multispecies forests with diverse financial and ecological objectives, of which even-aged management is a special case. The methods extend Markov decision process (MDP) models in forestry (e.g., Buongiorno 2001; Lembersky and Johnson 1975; Martell 1980; Zhou and Buongiorno 2011). MDP models have been effective in modeling forest ecosystems subject to the uncertainty of forest growth and wood markets. In particular, linear programming solutions of MDP models allow for multiple economic and ecological goals with the objective function and (or) constraints (Buongiorno and Zhou 2015). However, most of the published applications consider only the expected values of criteria and ignore their variances. Applications of risk-sensitive MDP models in natural resource management are rare. Ermon et al. (2011) suggest their application in fishery management. In forestry, Brunette et al. (2015) deal with intertemporal utility but without considering variations in the returns.

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Table 1. Low and high levels of basal area by tree species and size used to define stand states.

Basal area	Pines ($m^2 \cdot ha^{-1}$)			Hardwoods ($m^2 \cdot ha^{-1}$)		
	Pulpwood	Small sawtimber	Large sawtimber	Pulpwood	Small sawtimber	Large sawtimber
Low (0)	≤2.5	≤4.3	≤3.6	≤2.9	≤1.2	≤1.7
High (1)	>2.5	>4.3	>3.6	>2.9	>1.2	>1.7

The objective of this study was to propose models of risk-sensitive MDP models extending the linear-programming (LP) formulation of risk-neutral MDP models in multiple-use forestry. Two risk measures were used: the variance and the certainty equivalent of the financial or ecological criteria. One model maximized the expected value of a criterion while keeping its variance at a satisfactory level or, symmetrically, minimized the variance of a criterion while constraining the expected value of other objectives. The certainty equivalent formulation took the weighted average of the expected reward and its variance as an approximation of the expected utility function to explore various risk attitudes. The case study of uneven-aged loblolly pine and hardwood forests in the southern United States showed that compared with risk neutrality, financial risk aversion led to shorter cutting cycles, reduced expected annual financial returns and production, and also lowered the expected diversity of tree species and size, stand basal area, stored CO_2e, and old growth area.

Methods and data

Constrained variance

Most applications of MDP models in forestry have assumed risk neutrality of the decision maker, interpreted with an objective function that expresses the expected undiscounted or discounted value of a particular criterion. Not discounting or, equivalently, assuming a zero discount rate can been justified as treating present and future generations equally, encouraging long-term investments, and as a precaution against irreversible decisions (Fischer and Kuzivanova 2014; Goklany 2002). For example, the policy that maximizes the undiscounted expected financial annual return over an infinite horizon can be obtained by solving the following linear program (Hillier and Lieberman 2010, p. 913; Manne 1960):

$$(1)\quad \begin{aligned} &\max_{z_{ik}} E(R) = \sum_{i,k} z_{ik} r_{ik} \\ &\text{subject to} \\ &\sum_{k} z_{jk} - \sum_{i,k} z_{ik} p(j|i,k) = 0 \quad \forall j \\ &\sum_{i,k} z_{ik} = 1 \\ &\text{and} \\ &z_{ik} \geq 0 \quad \forall i,k \end{aligned}$$

where $E(R)$ is the expected annual return (dollars per hectare per year; expressed in US dollars throughout); r_{ik} is the immediate return in state i with decision k, where state i is a combination of stand and market states and decision k is a harvest that changes the stand state; z_{ik} is the probability of state i and decision k; and $p(j|i, k)$ is the probability of ending in state j in one year, given state i and decision k.

After solving this problem (eq. 1), the best policy, i.e., the decision for each stand and market state, is

$$(2)\quad X_{ik} = \frac{z_{ik}}{\sum_{k} z_{ik}}$$

where X_{ik} is the probability of decision k conditional on state i, which is always 0 or 1 (Hillier and Lieberman 2010, p. 787).

Risk aversion means that, in addition to the expected value of the annual return, the decision maker is also sensitive to its variance. Specifically, for two policies with equal expected outcomes, risk-averse decision makers prefer the alternative that has the lowest variance. In this application, the variance, V, of a stream of annual returns per unit of land area, R, was

$$(3)\quad V(R) = E(R^2) - [E(R)]^2$$

where E was the expected value. With the notations used in model 1 (eq. 1), the variance of the annual return became

$$(4)\quad V(R) = \sum_{i,k} z_{ik}(r_{ik})^2 - \left[\sum_{i,k} z_{ik} r_{ik}\right]^2$$

According to Markowitz's portfolio selection theory (Markowitz 1952), efficient harvest policies for rational risk-averse decision makers must be such that they minimize the risk (expressed by the variance) for any expected value of returns, $E(R)^*$, or equivalently, the decisions must maximize the expected value for any particular level of risk, $V(R)^*$. Such a policy was obtained by modifying model 1 into the following quadratic program:

$$(5)\quad \begin{aligned} &\max_{z_{ik}} E(R) = \sum_{i,k} z_{ik} r_{ik} \\ &\text{subject to} \\ &\sum_{k} z_{jk} - \sum_{i,k} z_{ik} p(j|i,k) = 0 \quad \forall j \\ &\sum_{i,k} z_{ik} = 1 \\ &z_{ik} \geq 0 \quad \forall i,k \\ &\text{and} \\ &\sum_{i,k} z_{ik}(r_{ik})^2 - \left[\sum_{i,k} z_{ik} r_{ik}\right]^2 \leq V(R)^* \end{aligned}$$

Solving model 5 (eq. 5) parametrically with increasing values of the variance $V(R)^*$ defined the efficient frontier and the corresponding set of solutions such that no other solution existed with a higher expected annual return and the same variance of return.

Symmetrically, the same efficiency frontier could be obtained by switching objective function and constraint in model 5 to minimize variance of returns subject to a constraint on their expected value.

Multiple constraints

Model 5 can be modified and extended in several ways by changing objective function and (or) constraints. For example, the policy that minimized the risk (variance) of financial annual returns, subject to lower bounds on the expected value of multiple criteria, was obtained by solving

Table 2. Price index (I_t) defining the market states, mean price level by market state and timber category, and transition probability between market states.

	Market state		
	Low	Medium	High
Price index	I_t<26.7	26.7<I_t≤38.8	I_t>38.8
Pines price ($·t⁻¹)			
Large sawtimber	28.6	39.5	54
Small sawtimber	14.3	19.8	27
Pulpwood	10.2	10.2	10.2
Hardwoods price ($·t⁻¹)			
Large sawtimber	16.5	20.3	25.7
Small sawtimber	8.3	10.2	12.9
Pulpwood	6.1	6.1	6.1
	Annual transition probability between states		
Market state	Low	Medium	High
Low	0.71	0.25	0.04
Medium	0.27	0.52	0.21
High	0.04	0.21	0.75

$$\min_{z_{ik}} V(R) = \sum_{i,k} z_{ik}(r_{ik})^2 - \left[\sum_{i,k} z_{ik} r_{ik}\right]^2$$

subject to

$$(6) \quad \sum_k z_{jk} - \sum_{i,k} z_{ik} p(j|i,k) = 0 \quad \forall j$$

$$\sum_{i,k} z_{ik} = 1$$

$$z_{ik} \geq 0 \quad \forall i,k$$

and

$$\sum_{i,k} z_{ik} c_{ikm} \geq C_m^* \quad \forall m$$

where C_m^* was the lower bound acceptable for the expected value of criterion m. Alternatively, a symmetric version of model 6 (eq. 6) would maximize the expected value of a criterion, subject to constraints on the variance of multiple objectives.

Certainty equivalent

An alternative approach to maximizing the expected value of a criterion subject to constrained variance, or minimizing variance subject to constrained expected value, was to use an objective function that combined expected value and variance. A classical utility function of this kind is the certainty equivalent exponential utility function, of which the quadratic approximation is (Ding et al. 2009; Freund 1956; Levitt and Ben-Israel 2001) as follows:

$$(7) \quad U(X) = E(X) - \frac{\beta}{2} V(X)$$

where U is the utility of a particular criterion X of expected value E and variance V. In our context, for the financial criterion, $U(X)$ was the amount of "certain" annual income that the decision maker would be willing to receive in exchange for the returns expected from the forest. The parameter β indicates the risk attitude of the decision maker. A positive β means risk aversion, i.e., a conflict between expected value and variance, whereas a negative β suggests a risk-seeking attitude in which, for a given expected value, additional variance increases utility, and $\beta = 0$ expresses risk neutrality in which only expected value matters. β is a constant measure of risk preference assumed independent of the wealth of the decision maker.

Fig. 1. Efficient frontier for risk aversion in selected criteria. Each point on the frontier is the maximum annual expected value of the criterion subject to an upper bound on its standard deviation.

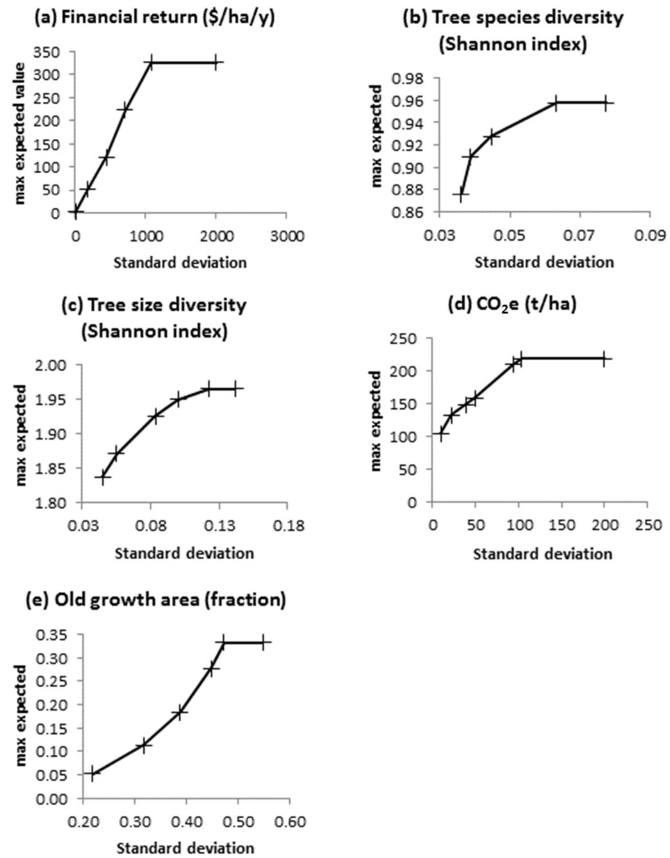

Table 3. Expected value and standard deviation of management criteria for financially risk-averse or risk-neutral decision makers.

		Risk averse		Risk neutral	
Criterion	Unit	Expected value	Standard deviation	Expected value	Standard deviation
Return	$·ha⁻¹·year⁻¹	140.1	500	326.0	1079
Species diversity	Index	0.83	0.10	0.92	0.08
Size diversity	Index	1.83	0.06	1.84	0.08
Basal area	m³·ha⁻¹	14.5	2.32	15.9	3.0
Stored CO_2e	t·ha⁻¹	111.0	31.64	127.8	31.2
Old growth	Fraction	0.00	0.01	0.01	0.12
Production	m³·ha⁻¹·year⁻¹	5.3	14.46	6.3	18.1
Cutting cycle	Years	6.8		7.9	

Note: The risk-averse solution maximized expected annual return subject to a standard deviation less than or equal to $500·ha⁻¹·year⁻¹, while the risk-neutral solution maximized unconstrained expected annual return.

In the context of the problem described by the MDP model 1, the utility of the stream of annual financial returns became

$$(8) \quad U(X) = \sum_{i,k} z_{ik} r_{ik} - \frac{\beta}{2}\left(\sum_{i,k} z_{ik}(r_{ik})^2 - \left[\sum_{i,k} z_{ik} r_{ik}\right]^2\right)$$

Levitt and Ben-Israel (2001) use the objective function of eq. 7 to explicitly model risk attitude in MDP models using the Bellman optimality principle and its related dynamic programming numerical solutions. Here instead we pursued the linear programming approach expressed by model 1, which, with the certainty equivalent objective (eq.8), became

Table 4. Best decision by stand and market state (low, medium, or high) for financially risk-averse or risk-neutral decision makers.

Stand state no.	Stand composition[a]	Decision[b] Risk averse Low	Medium	High	Risk neutral Low	Medium	High
1	000,000	—	—	—	—	—	—
2	000,001	—	—	1	—	—	—
3	000,010	—	—	1	—	—	—
4	000,011	2	2	3	—	—	—
5	000,100	—	—	—	—	—	—
6	000,101	2	2	5	5	5	5
7	000,110	3	3	5	—	—	—
8	000,111	4	7	7	4	4	7
9	001,000	—	—	—	—	1	1
10	001,001	2	9	9	—	2	2
11	001,010	3	—	9	—	3	3
12	001,011	4	11	11	—	4	4
13	001,100	5	9	9	—	5	5
14	001,101	6	10	13	13	5	5
15	001,110	11	11	11	—	7	7
16	001,111	8	15	15	—	4	7
17	010,000	1	1	—	—	—	1
18	010,001	2	2	17	—	—	2
19	010,010	3	3	17	—	—	3
20	010,011	4	4	19	—	19	4
21	010,100	5	5	17	—	—	5
22	010,101	6	6	21	21	21	5
23	010,110	7	7	21	19	—	7
24	010,111	8	8	23	20	23	7
25	011,000	9	9	—	17	17	1
26	011,001	10	10	25	18	18	2
27	011,010	11	11	25	19	19	3
28	011,011	12	12	27	20	20	4
29	011,100	13	13	25	21	21	5
30	011,101	14	14	29	—	—	—
31	011,110	15	15	27	19	23	7
32	011,111	16	16	31	20	23	7
33	100,000	1	1	1	—	—	—
34	100,001	2	2	33	2	33	33
35	100,010	3	3	3	—	—	—
36	100,011	—	35	35	—	—	—
37	100,100	5	5	5	5	5	5
38	100,101	6	6	37	5	5	5
39	100,110	35	35	35	35	35	35
40	100,111	36	36	39	36	36	36
41	101,000	—	—	9	—	33	33
42	101,001	34	41	41	41	33	33
43	101,010	11	11	41	—	35	35
44	101,011	36	43	43	36	36	36
45	101,100	41	41	41	—	5	5
46	101,101	—	45	—	—	5	5
47	101,110	15	15	15	—	35	35
48	101,111	44	47	47	—	36	36
49	110,000	33	33	17	—	—	33
50	110,001	34	34	49	18	49	33
51	110,010	35	35	19	—	—	35
52	110,011	36	36	51	—	—	36
53	110,100	37	37	21	21	21	5
54	110,101	—	—	—	—	—	5
55	110,110	39	39	23	—	—	35
56	110,111	—	—	—	52	52	36
57	111,000	41	41	25	49	49	33
58	111,001	—	—	—	—	—	—
59	111,010	43	43	27	51	51	35
60	111,011	44	44	59	52	52	36
61	111,100	45	45	29	21	21	5
62	111,101	46	—	61	54	54	5
63	111,110	47	47	31	55	55	35
64	111,111	48	48	63	52	52	36
No. of harvest decisions		53	53	55	30	43	53
Total no. of harvest decisions by risk solution		161			126		

Table 4 (*concluded*).

Note: The risk-averse solution maximized expected annual return subject to a standard deviation less than or equal to \$500·ha⁻¹·year⁻¹, whereas the risk-neutral solution maximized unconstrained expected annual return.

[a]Basal area in pulpwood, small sawtimber, and large saw timber of pines (first three digits) and hardwoods (last three digits): 1 = higher than current average, 0 = lower than current average.

[b]Stand state no. resulting from the best harvest decision; "—" indicates no harvest.

$$
\begin{aligned}
&\max_{z_{ik}} U(R) = \sum_{i,k} z_{ik} r_{ik} - \frac{\beta}{2}\left(\sum_{i,k} z_{ik}(r_{ik})^2 - \left[\sum_{i,k} z_{ik} r_{ik}\right]^2\right) \\
&\text{subject to} \\
&\sum_{k} z_{jk} - \sum_{i,k} z_{ik}\, p(j|i,k) = 0 \quad \forall j \qquad (9) \\
&\sum_{i,k} z_{ik} = 1 \\
&\text{and} \\
&z_{ik} \geq 0 \quad \forall i,k
\end{aligned}
$$

With $\beta = 0$, model 9 (eq. 9) reduces to model 1, maximizing expected annual returns to express risk neutrality.

Stand growth and market models

The forests considered in the following applications were uneven-aged mixed loblolly pines and hardwoods in the southern United States. We used the Markov chain model of forest stand growth in Zhou and Buongiorno (2006). In this model, each stand state is represented by a string of six digits such as (010,001), where the first three stand for the basal area (0 for low, 1 for high; defined in Table 1) of pulpwood, small sawtimber, and large sawtimber of pines, and the last three are for hardwoods. Six tree classes with two levels of basal area make up $2^6 = 64$ possible stand states. The yearly harvest changes the stand state instantaneously by reducing the basal area in one or more tree groups. Furthermore, to take catastrophic risk into consideration, Zhou and Buongiorno (2006) assume a mean annual rate of hurricanes of category 4 and above to be 0.025 based on historical records. Due to limited information to quantify vulnerability of forest states to hurricanes, they further assume that regardless of the current state, such a hurricane reduces the stand basal area in the six tree groups to its lowest level (state 1). Consequently, the transition probability from any state s to state 1 is changed to $p(1|s) \times (1 - 0.975) + 0.025$ and the other probabilities become $p(s'|s) \times 0.975$ for any state s' other than state 1.

For simplicity, to reduce the size of the problem and because the data showed a positive correlation between the price of softwoods and hardwoods, the market state was categorized as low, medium, or high according to the level of a single stumpage price index. The index was the weighted average of the real stumpage price of softwoods and hardwoods from 1977 to 2014 (data for 1977–2014 from TimberMart-South, Frank W. Norris Foundation, Athens, Georgia; available from http://www.timbermart-south.com/), with the growing stock volume serving as weights. Table 2

Table 5. Expected value and standard deviation of management criteria for decision makers minimizing the variance of financial annual returns subject to reaching at least half of the maximum unconstrained expected value of all criteria.

Criterion	Unit	Expected value (1)	Standard deviation	Maximum unconstrained expected value (2)	(1)/(2)
Financial return	$\$\cdot ha^{-1}\cdot year^{-1}$	163	586	326	0.50
Tree species diversity	Index	0.89	0.09	0.96	0.93
Tree size diversity	Index	1.86	0.10	1.97	0.95
Basal area	$m^3\cdot ha^{-1}$	16.0	3.4	21.7	0.74
CO_2e	$t\cdot ha^{-1}$	122.9	30.0	218.6	0.56
Old growth	Fraction	0.17	0.37	0.33	0.50
Production	$m^3\cdot ha^{-1}\cdot year^{-1}$	5.1	15.3	7.5	0.68
Cutting cycle	Years	8.4			

shows the definition of the three market states (low, medium, and high) based on this index, the price of softwoods and hardwoods in each state, and the annual transition probabilities between states based on the data from 1977 to 2014. With this assumption, the prices of hardwoods and softwoods and the prices of large and small sawtimber were perfectly correlated.

With 64 stand states and three price levels, there were 192 stand–market states. The transition probability between each pair of stand–market states was the product of the transition probabilities between the stand states and between the market states.

Management criteria

The expected immediate financial reward depended on the volume of harvest due to a decision and on the market state. The other criteria considered in the following applications were the tree size and species diversity measured with Shannon's index, the stand basal area ($m^3\cdot ha^{-1}$), the CO_2 equivalent (CO_2e) stored in trees ($t\cdot ha^{-1}$), the fraction of the landscape in old-growth state over a large forest area, and the expected cutting cycle in years (Zhou and Buongiorno 2006). For each criterion, the expected value and the variance were computed from the probability of a stand–market state and decision, z_{sd}, and the level of the criterion resulting from the decision, r_{sd}.

Results

Risk aversion

Model 5 was applied to calculate the efficiency frontiers for different criteria, i.e., the maximum expected value of a criterion subject to prespecified upper bounds on its variance (Fig. 1). For example, with the financial objective (Fig. 1*a*), for decision makers unwilling to tolerate more than $\$500\cdot ha^{-1}\cdot year^{-1}$ of standard deviation in annual returns, the highest possible expected annual return was $\$140\cdot ha^{-1}\cdot year^{-1}$ or, symmetrically, an average return of $\$140\cdot ha^{-1}\cdot year^{-1}$ could not be obtained with a standard deviation less than $\$500\cdot ha^{-1}\cdot year^{-1}$.

For financial returns, the efficiency frontier was approximately linear for standard deviations between \$0 and $\$1079\cdot ha^{-1}\cdot year^{-1}$. Within that range, increasing the expected return by $\$1\cdot ha^{-1}\cdot year^{-1}$ required accepting an increase in standard deviation of approximately $\$3.10\cdot ha^{-1}\cdot year^{-1}$. Allowing the standard deviation to exceed $\$1079\cdot ha^{-1}\cdot year^{-1}$ could not increase the annual expected return beyond $\$326\cdot ha^{-1}\cdot year^{-1}$, the maximum achievable with unconstrained variance.

The efficiency frontiers in Fig. 1 can be used by decision makers to express their risk aversion. Assume, for example, using Fig. 1*a*, that they set their upper bound on financial risk at $\$500\cdot ha^{-1}\cdot year^{-1}$ of standard deviation, with a corresponding maximum expected annual return of $\$140\cdot ha^{-1}\cdot year^{-1}$. Table 3 shows the effects of this level of financial risk aversion on the expected value and standard deviation of other criteria next to their level for financially risk-neutral decision makers. The expected values of annual return and production were markedly lower with financial risk aversion than with risk neutrality. This was obtained with an average cutting cycle approximately one year shorter with the financially risk-averse solution than with the risk-neutral one. This shorter cutting cycle induced lower expected values of the ecological criteria, especially species diversity, basal area, and stored CO_2e. In this example, higher diversity of tree size and species could not reduce the variance of returns due to the perfect positive correlation between the prices of softwoods and hardwoods and the prices of large and small sawtimber (Table 2). The corresponding financially risk-averse and risk-neutral policies, i.e., the decisions depending on the stand and market state, are given in Table 4. With risk aversion, 161 stand–market states called for a harvest (53 with low or medium price, and 54 with high price) versus 126 with risk neutrality (30 with low price, 43 with medium price, and 53 with high price)[1].

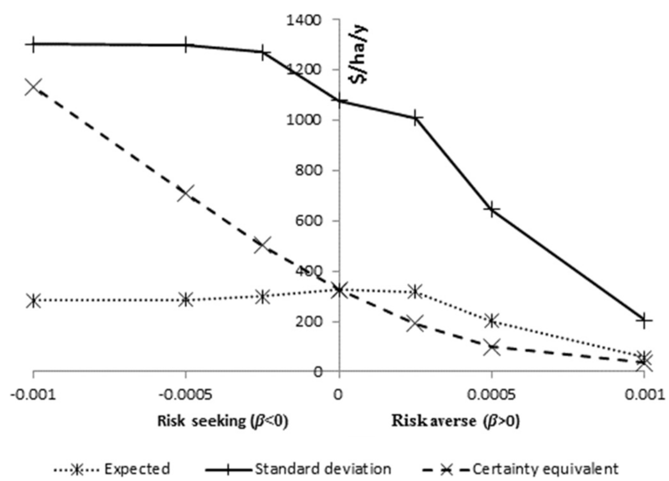

Fig. 2. Effects of the risk-aversion parameter β of the certainty equivalent model on the expected value, standard deviation, and certainty equivalent value of annual financial returns: $\beta > 0$ indicates risk aversion, $\beta < 0$ indicates risk seeking, and $\beta = 0$ indicates risk neutrality of decision maker.

[1]Due to numerical errors and the presence of nonlinear constraints, a few of the decision variables, X_{ik}, in eq. 2 were not 0 or 1, but fractions close to 0 or 1, which were rounded off to obtain the policy Tables 4 and 7.

Table 6. Expected value and standard deviation of management criteria, according to financial risk attitude, obtained by maximizing certainty equivalent annual income.

		Risk averse β = 0.0005		Risk seeking β = −0.0005	
Criterion	Unit	Expected value	Standard deviation	Expected value	Standard deviation
Financial return	$\$\cdot ha^{-1}\cdot year^{-1}$	202	645	287	1301
Certainty equivalent	$\$\cdot ha^{-1}\cdot year^{-1}$	98		710	
Tree species diversity	Index	0.88	0.10	0.89	0.09
Tree size diversity	Index	1.82	0.06	1.86	0.10
Basal area	$m^3\cdot ha^{-1}$	14.6	2.2	17.2	4.4
CO_2e	$t\cdot ha^{-1}$	116	26.2	147	66.7
Old growth	Fraction	0.00	0.02	0.02	0.15
Production	$m^3\cdot ha^{-1}\cdot year^{-1}$	6.1	16.5	4.7	20.3
Cutting cycle	Years	6.6		14.2	

Multiple constraints

As an example of application of risk aversion with multiple constraints (model 6), Table 5 shows the results of the solution that minimized the variance of financial annual returns subject to keeping the expected value of all criteria at least at 50% of their unconstrained maxima. In the optimal solution, annual returns and the fraction of forest in old growth were at their 50% lower bound, while other criteria exceeded it. In particular, diversity of tree species and size were at 93% and 95% of their maximum unconstrained value, respectively. Compared with the risk-averse solution in Table 3 that maximized annual returns subject only to a standard deviation of $500·ha^{-1}·year^{-1}, most expected values were higher. The only exception was expected production, which was marginally lower with the multiple constraints (Table 5) than with the single constraint of $500·ha^{-1}·year^{-1} on the standard deviation of annual returns (Table 3). The expected fraction of old growth was markedly higher with the multiple constraints (Table 3), as was the standard deviation.

Certainty equivalence

Application of the certainty equivalent model 9 requires choosing a particular value of the risk-aversion parameter, β. For the financial criterion, Fig. 2 shows how the expected value, standard deviation, and the certainty equivalent annual returns changed when the certainty equivalent $U(X)$ was maximized with values of β varying from −0.001 (risk seeking) to 0.001 (risk aversion).

With risk neutrality (β = 0), the certainty equivalent return was the same as the expected annual return of $326·ha^{-1}·year^{-1}, and a corresponding standard deviation was $1079·ha^{-1}·year^{-1}. Positive values of β indicating rising risk aversion steadily decreased expected value, variance, and certainty equivalent annual return. With β = 0.001, the expected return was 82% less than with risk neutrality, the standard deviation was 81% less, and the certainty equivalent return was 89% less.

In contrast, progressively negative values of β characteristic of risk seeking decreased expected returns while increasing the variance and the certainty equivalent return. For $\beta \leq -0.005$, the expected value remained approximately constant at 13% less than with risk neutrality, and the standard deviation was 21% more than with risk neutrality. In contrast, the certainty equivalent income increased steadily from $326·ha^{-1}·year^{-1} with risk neutrality to $1134·ha^{-1}·year^{-1} for a risk seeker with β = −0.001.

Table 6 shows the effects on all management criteria of maximizing the certainty equivalent annual return with moderate levels of financial risk aversion (β = 0.0005) and financial risk seeking (β = −0.0005). The certainty equivalent financial annual return was seven times higher with risk seeking than with risk aversion, the standard deviation was twice as large, and the expected value was 42% higher. The expected diversity of tree species and size were similar, but the standard deviation of size diversity was 50% higher with risk seeking. Expected basal area and CO_2e were 20% to 30% higher with risk seeking and their standard deviations were 2 to 2.5 times higher. With risk aversion, there was, on average, practically no part of the landscape left in old growth compared with 2% with risk seeking, but with a standard deviation seven times as large. Expected annual production was 22% less, but standard deviation was 23% more with risk seeking. This and the expected cutting cycle that was more than twice as long with risk seeking suggested that the lesser volume harvested contained, on average, more high-value sawlogs than pulpwood.

The policies that led to these results are in Table 7. Overall, with the risk-averse policy (β = 0.0005), 144 stand–market states called for a harvest (47 at low market level, 48 at medium, and 49 at high) compared with only 58 with the risk-seeking policy (12 at low market, 8 at medium, and 38 at high), in agreement with the longer cutting cycle expected with the risk-seeking policy (Table 6).

Summary and conclusion

Two methods were used to extend MDP models that classically deal with the expected value of forestry decision criteria to also recognize their variance. One approach, based on Markowitz's portfolio theory, consisted of maximizing the expected value of a criterion subject to an upper bound on the variance or, symmetrically, minimizing the variance subject to a lower bound on the expected value. This paper showed how this principle could be applied to MDP models with criteria such as the annual return per unit of forest land or the amount of CO_2e stored in a stand of trees. The approach used the central variable of the linear-programming MDP formulation, the probability of a decision given a system state (here combining the state of a forest stand and the state of the market). The other method used the certainty equivalent criterion, essentially a weighted average of the expected value and variance.

The two approaches were applied to data for mixed softwood–hardwood forests in the southern United States. Markowitz's constrained method gave the efficient expected value–variance frontier for annual financial returns and other criteria. The effects of choosing different points on the financial efficiency frontier were derived. They showed that higher financial risk aversion besides reducing expected annual financial returns and production also reduced expected species and size diversity, stand basal area, stored CO_2e, old-growth area, and length of cutting cycle. This model was extended to predict the effect of multiple constraints on the expected values of ecological criteria. In practice, it can be challenging to set specific bounds of multiple, often noncommensurable, conflicting objectives that accurately represent decision makers' preference (Tamiz et al. 1998). Buongiorno and Zhou (2017) propose methods combining goal programming and MDP that minimize deviations from desired levels of multiple objec-

Table 7. Decisions by stand and market state (low, medium, or high) that maximized the certainty equivalent annual returns for risk-averse and risk-seeking decision makers.

		Decision[b]					
		Risk averse (β = 0.0005)			Risk seeking (β = −0.0005)		
Stand state no.	Stand composition[a]	Low	Medium	High	Low	Medium	High
1	000,000	—	—	—	—	—	—
2	000,001	—	—	—	—	—	—
3	000,010	—	—	—	—	—	—
4	000,011	—	2	3	—	—	—
5	000,100	—	—	—	—	—	—
6	000,101	2	5	5	2	—	—
7	000,110	—	5	5	—	—	—
8	000,111	4	7	7	4	4	4
9	001,000	1	—	—	—	—	1
10	001,001	2	—	9	—	—	1
11	001,010	3	9	9	9	—	3
12	001,011	4	10	11	—	4	3
13	001,100	5	—	9	—	—	5
14	001,101	6	10	13	10	—	5
15	001,110	7	—	13	—	—	5
16	001,111	8	15	15	—	—	5
17	010,000	—	1	1	—	—	—
18	010,001	2	2	2	—	—	—
19	010,010	—	3	3	—	—	—
20	010,011	4	4	4	—	—	—
21	010,100	5	5	5	—	—	—
22	010,101	6	6	6	18	18	18
23	010,110	7	7	7	—	—	19
24	010,111	8	8	8	20	20	—
25	011,000	17	9	9	—	—	1
26	011,001	18	10	10	—	—	1
27	011,010	19	11	11	—	—	3
28	011,011	20	12	12	—	—	3
29	011,100	21	13	13	—	—	5
30	011,101	—	—	—	—	—	5
31	011,110	23	15	15	—	—	5
32	011,111	—	—	—	28	—	5
33	100,000	—	—	—	—	—	—
34	100,001	2	2	33	—	—	—
35	100,010	—	—	33	—	—	—
36	100,011	—	—	—	—	—	—
37	100,100	5	5	5	—	—	—
38	100,101	6	6	37	—	—	—
39	100,110	35	35	35	35	35	35
40	100,111	36	36	36	36	36	36
41	101,000	33	—	—	—	—	33
42	101,001	34	41	41	—	—	33
43	101,010	35	41	41	—	—	35
44	101,011	36	43	43	—	36	36
45	101,100	37	41	41	—	—	5
46	101,101	38	45	—	—	—	5
47	101,110	39	43	41	—	—	35
48	101,111	40	44	47	—	—	36
49	110,000	33	33	33	—	—	—
50	110,001	18	34	34	—	—	—
51	110,010	35	35	35	—	—	—
52	110,011	36	36	36	—	—	—
53	110,100	21	37	37	—	—	—
54	110,101	—	38	—	—	—	—
55	110,110	51	39	39	—	—	—
56	110,111	52	40	40	52	52	52
57	111,000	49	41	41	—	—	1
58	111,001	—	—	—	—	—	1
59	111,010	51	43	43	—	—	35
60	111,011	52	44	44	—	—	36
61	111,100	—	45	—	—	—	5
62	111,101	—	—	—	58	—	5
63	111,110	55	47	47	—	—	5
64	111,111	56	48	48	60	—	5
No. of harvest decisions		47	48	49	12	8	38
Total no. of harvest decisions by risk solution		144			58		

[a]Basal area in pulpwood, small sawtimber, and large saw timber of pines (first three digits) and hardwoods (last three digits): 1 = higher than current average, 0 = lower than current average.

[b]Stand state no. resulting from the best harvest decision; "—" indicates no harvest.

tives for risk-neutral landowners. It would be fruitful to adapt such methods for risk-sensitive decision makers. The methods proposed here could also be used with finer data such as prices distinguished by tree species or size to study the effect of differentiation on risk, if warranted by strong negative correlations, and at the cost of an increase in model size. Improvements could also be made to model the effect of catastrophes, their probability, the resulting stand states, and the attendant costs and benefits of harvesting and restoring a stand after such events.

Applications of the certainty equivalent model revealed the sensitivity of the financial criterion to the values of the risk-aversion parameter β. With moderate values of β, financial risk seeking led to higher expected values of most criteria but larger standard deviations, with cutting cycles twice as long as under risk aversion.

There is scant reason to prefer either of the two approaches investigated here. For strictly risk-averse decision makers, constrained variance is more direct. For a single criterion in particular, the expected value–variance efficiency frontier is helpful in choosing a trade-off point between the two. The certainty equivalent method has the advantage of dealing with risk seeking as well as risk aversion. Although it was applied here only to financial returns, certainty equivalent values could similarly be developed for other criteria. Admittedly, a large percentage of landowners, especially private ones, are usually considered risk averse (Henderson 2007; Lönnstedt and Svensson 2000). Nevertheless, it is still worth studying risk-seeking behavior as risk attitude is known to change and has profound effects on timber harvests (Uusivuori 2002). Moreover, risk seeking may be appropriate to depict risk preference of some absentee landowners who are financially less dependent on the land than those living on the land (Petrzelka et al. 2013, p. 157).

Regardless of method, in practical decision making and especially in multiple-use forests, multiple criteria must be considered simultaneously. In that respect, both the constrained variance and the certainty equivalent approach combined with the linear programming formulation of MDP models are compelling for the ease of introducing multiple constraints on expected value or variance of criteria, or both. Although the programs become nonlinear with variance-dependent objective function or constraints, they are quadratic and thus simple to solve numerically. The two approaches could also be augmented with goal programming, allowing versatile formulations of noncommensurable and conflicting objectives while fulfilling decision makers' needs of optimizing or satisficing (Buongiorno and Zhou 2017). More difficult would be dealing with the variance of discounted rather than average criteria (Ruszczyński 2010; Shen et al. 2013), a problem worthy of further research.

Inevitably, the proposed methods are subject to the inherent limitations of MDP models, especially the "curse of dimensionality," which forces some simplifications to render models more computationally tractable. In addition, variance, the measure of risk used in the first method, has been criticized for not being "coherent" (Artzner et al. 1999), while the certainty equivalent model assumed constant risk preference thus ignoring wealth effects (Friedman and Savage 1948). Future efforts should be directed towards solving these issues in the MDP framework.

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